AVERAGE ALONG HOLOMORPHIC CURVES

CHAN WOO YANG

Abstract. Averaging operators carry certain types of improving properties and investigating optimal improving properties of averaging operators is one of the main issues in harmonic analysis. The maximal amount of integrability and differentiability of functions which are bettered by averaging operators has been known to depend on the geometry of surfaces along which the integration is performed for the operators to be defined.
Let $z = x + iy \in \mathbb{C}$ and let $\gamma(z) = u(z) + iv(z)$ be a holomorphic function in an open set $\Omega \in \mathbb{C}$. We consider a holomorphic curve Γ in $\mathbb{C}^2 = \mathbb{R}^4$ of the form

$$\Gamma(z) = (z, \gamma(z)),$$

which is a graph of the holomorphic function γ. We consider averaging operators $\mathcal{R}$ along the holomorphic curve Γ defined by

$$\mathcal{R}f(w) = \int_{\mathbb{C}} f(w - \Gamma(z))\chi(z)dz,$$

where $w = (w_0, w_1) \in \mathbb{C}^2$, χ is a smooth cut-off function supported in a bounded neighborhood U of the origin, and $dz = dx\,dy$.
In this talk we discuss on sharp L^p improving estimates for averaging operators $\mathcal{R}$ along holomorphic curves Γ in $\mathbb{C}^2$.