# 7th Workshop on PDEs of Mathematical Physics

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**List of speakers**

김 도윤 (Korea Univ.)
이 지훈 (Chung-Ang Univ.)
고 은경 (SNU)
김 용정 (KAIST)
배 형옥 (Ajou Univ.)
석 진명 (Kyunggi Univ.)
양 민석 (KIAS)
최 선호 (NIMS)
김 두규 (Sogang Univ.)
홍 석현 (POSTECH)

**Date** || Jan 13 (Wed) – Jan 15 (Fri), 2016
**Place** || UNIST, Ulsan, Korea

**Organized by**
권 봉석 (UNIST)
배 한택 (UNIST)
최 규동 (UNIST)
석 진명 (Kyunggi Univ.)

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Department of Mathematical Sciences, UNIST
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## Schedules

| | Jan 13 (Wednesday)<br>**EB1-E101** | Jan 14 (Thursday)<br>**BAB-106** | Jan 15 (Friday)<br>**BAB-106** |
|---|---|---|---|
| 09:30~ 10:20 | Opening group discussion<br>**BAB-806-13** | Chair: 정 소연**BAB-106**<br>고 은경<br>(SNU) | Chair: 유 민하**BAB-106**<br>최 선호<br>(NIMS) |
| 10:20~ 10:40 | | Q/A & discussion | Q/A & discussion |
| 10:40~ 11:20 | | 홍 석현<br>(POSTECH) | 김 두규<br>(Sogang Univ.) |
| 11:20~ 12:10 | | 김 용정<br>(KAIST) | 양 민석<br>(KIAS) |
| 12:10~12:30 | | Q/A & discussion | Discussion & closing remark |
| 12:30~ 14:00 | | **Lunch Time** | |
| 14:00~ 14:50 | Chair: 김 현석**EB1-E101**<br>김 도윤 #1<br>(Korea Univ.) | Chair: 윤 석배**BAB-106**<br>김 도윤 #2<br>(Korea Univ.) | |
| 15:00~ 15:50 | 이 지훈 #1<br>(Chung-Ang Univ.) | 이 지훈 #2<br>(Chung-Ang Univ.) | |
| 16:00~ 16:50 | 석 진명<br>(Kyunggi Univ.) | 배 형옥<br>(Ajou Univ.) | |
| 16:50~18:00 | Q/A & discussion | Q/A & discussion | |
| 18:00~ 20:00 | **Dinner** | **Banquet** | |

# Time-asymptotic interaction of flocking particles and an incompressible viscous fluid, and its simmulation

Hyeong-Ohk Bae

Jan. 13 – 15, 2015

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**Abstract**

We have presented a new coupled kinetic-fluid model for the interactions between Cucker-Smale(C-S) flocking particles and fluid recently. Our coupled system consists of the kinetic Cucker-Smale equation and the incompressible Navier-Stokes equations, and these two systems are coupled through the drag force. For the proposed model, we provide numerical simulations to look at their behavior.

# Kinetic description for sheath formations.

Sun-Ho Choi

Jan. 13 – 15, 2015

NIMS
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**Abstract**

Sheath formation is a kind of phase state of gas. Shortly, phase transition diagram is solid-liquid-gas-plasma-sheath. In this talk, I would like to present a kinetic description for sheath formations. The first topic is a stationary Vlasov-Poisson system representing ion density on symmetric container with centered negative charged ball. Here, the stationary Vlasov-Poisson system is equivalent to the Gelfand equation with Neumann boundary conditions. The Neumann boundary conditions come from mass conservation and electric charge conservation. In order to prove the existence of the solution to the Gelfand equation, we first transform this problem into terminal value problem. The second topic is planar sheath formation. This case also derives a stationary equation for 1D initial boundary value problem of a time evolution nonlinear Vlasov system.

# Stability of the self-similar shrinking solution of the evaporating-droplet problem

Seok Hyun Hong

Jan. 13 – 15, 2015

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**Abstract**

We study a free boundary problem that arises in models of the evaporation process. A phenomenon is considered as a singularity of the PDEs-based model. We prove stability of this singularity that represents the type-1 shrinking sphere of which the scaling exponent of the radius is 1/2. The main result reads as follows. The shrinking sphere is stable under small perturbations in $H^s$. Small initial-perturbations produce shifts of the singularity position in space and time, but not deviation from the singularity mechanism. Moreover, the solution tends to an ellipsoid determined by the initial shape and the capillary number as time approaches the singularity time.

# Mean oscillation estimates for elliptic and parabolic equations

Doyoon Kim

Jan. 13 – 15, 2015

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**Abstract**

When establishing the unique solvability of elliptic and parabolic equations/systems in Sobolev or Holder spaces, one useful approach is the mean oscillation estimate of solutions combined with well known theorems from harmonic analysis. In this talk we discuss how to obtain and use mean oscillation estimates of solutions to elliptic and parabolic equations/systems. We also discuss recent developments in the regularity theory of PDEs in Sobolev spaces.

# Very weak solutions of the stationary Navier-Stokes equations for an incompressible fluid past obstacles

Dugyu Kim Hyunseok Kim

**Abstract**

We consider the stationary motion of an incompressible Navier-Stokes fluid past obstacles in $\mathbb{R}^3$, subject to the given boundary velocity $v_b$, external force $f = \operatorname{div} F$ and nonzero constant vector $ke_1$ at infinity. Our main result is the existence of at least one very weak solution $v$ in $ke_1 + L^3(\Omega)$ for arbitrary large $F \in L^{3/2}(\Omega) + L^{12/7}(\Omega)$ provided that the flux of $v_b - ke_1$ on the boundary of each body is sufficiently small with respect to the viscosity $\nu$. The uniqueness of very weak solutions is proved by assuming that $F$ and $v_b - ke_1$ are suitably small. Moreover, we establish weak and strong regularity results for very weak solutions. In particular, our existence and regularity results enable us to prove the existence of a weak solution $v$ satisfying $\nabla v \in L^{3/2}(\Omega)$.

# Non-constant steady states driven by diffusion with spatial heterogeneity

Yong-Jung Kim

Jan. 13 – 15, 2015

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**Abstract**

Diffusion has been understood as a trivialization process that averages the initial variation and gives a constant steady state eventually. However, diffusion may produce nonconstant steady state if spatial heterogeneity is involved. In this talk we will see how such a diffusion is related to advection and cross-diffusion theory.

# A Three solution theorem for singular nonlinear elliptic boundary value problems

Eunkyung Ko

Jan. 13 – 15, 2015

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**Abstract**

In this talk, I present a three solution theorem for singular elliptic boundary value problems of the form $-\Delta u = \frac{f(u)}{u^\beta}$ in $\Omega$, $u = 0$ on $\partial\Omega$ where $\Omega$ is a bounded domain in $\mathbb{R}^N, N \geq 1$ with a smooth boundary $\partial\Omega$. Here $f : [0, \infty) \to [0, \infty)$ is a $C^1$ function in $[0, \infty)$ with $f(0) > 0$ and $\beta \in (0, 1)$. In particular if there exists two pair of sub-supersolutions $(\psi_1, \phi_1)$ and $(\psi_2, \phi_2)$ where $\psi_1 \leq \psi_2 \leq \phi_1, \psi_1 \leq \phi_2 \leq \phi_1$ with $\psi_2 \not\leq \phi_2$, and $\psi_2, \phi_2$ are strict sub and supersolutions, then we establish existence of three solutions $u_1, u_2$ and $u_3$ for the above boundary value problem such that $u_1 \in [\psi_1, \phi_2], u_2 \in [\psi_2, \phi_1]$ and $u_3 \in [\psi_1, \phi_1] \setminus ([\psi_1, \phi_2] \cup [\psi_2, \phi_1])$.

# Existence and Asymptotic Properties of Solutions to Some Chemotaxis models

Jihoon Lee

Jan. 13 – 15, 2015

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**Abstract**

First, we consider the existence and asymptotic properties of solutions to chemotaxis-Navier-Stokes equations :

$$\begin{cases} \partial_t n + u \cdot \nabla n - \Delta n = -\nabla \cdot (\chi(c) n \nabla c), \\ \partial_t c + u \cdot \nabla c - \Delta c = -k(c) n, \\ \partial_t u + u \cdot \nabla u - \Delta u + \nabla p = -n \nabla \phi, \quad \nabla \cdot u = 0 \end{cases} \tag{1}$$

where $c(t, x) : Q_T \to \mathbb{R}^+$, $n(t, x) : Q_T \to \mathbb{R}^+$, $u(t, x) : Q_T \to \mathbb{R}^d$ and $p(t, x) : Q_T \to \mathbb{R}$ denote the oxygen concentration, cell concentration, fluid velocity, and scalar pressure, respectively. The nonnegative function $k(c)$ denotes the oxygen consumption rate, and the nonnegative function $\chi(c)$ denotes chemotactic sensitivity. $\phi = \phi(x)$ denotes the potential function produced by different physical mechanisms.
Next, we consider the existence and asymptotic properties of solutions to coral spawning problem :
When divergence free $u(x, t) \in \mathbb{R}^d \times [0, \infty)$,

$$\begin{cases} \partial_t s + (u \cdot \nabla) s = \kappa_1 \Delta s - (se)^{\frac{q}{2}}, \quad s(x, 0) = s_0(x), \\ \partial_t e + (u \cdot \nabla) e = \kappa_2 \Delta e - (se)^{\frac{q}{2}}, \quad e(x, 0) = e_0(x). \end{cases} \tag{2}$$

Here, $s$ and $e$ denotes densities of sperm and egg gametes.
These are joint works with Professor Kyungkeun Kang(Yonsei U.) and Myeongju Chae(Hankyung National U.)

# Existence and regularity of nonlinear fractional scalar field equations

Jinmyoung Seok

Jan. 13 – 15, 2015

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**Abstract**

In this talk, we discuss about existence, regularity and qualitative properties of nonlinear fractional scalar field equations, which are fractional counterparts of nonlinear scalar field equations, studied by Berestycki and Lions in 1983. Under the almost optimal conditions, so-called "Berestycki-Lions" conditions, they proved the existence of classical least energy solutions of nonlinear scalar field equations. Our aim is to recover all the results by Berestycki and Lions for the fractional cases. We explain what have been recovered in their results and why some results are hard to recover.

# Hausdorff measure of boundary singular points in the magnetohydrodynamic equations

Minsuk Yang

Jan. 13 – 15, 2015

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**Abstract**

In this talk, we discuss about boundary singular points of suitable weak solutions to the three dimensional incompressible magnetohydrodynamic equations. By using the generalized Hausdorff measure we estimate the size of boundary singular points and present the improved range of powers of logarithmic factors.